MPHYCC-6

M.Sc. Sem II

Plasma Physics

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Perpendicular Wave Propagation

Let us now consider wave propagation, at arbitrary frequencies, perpendicular to the equilibrium magnetic field. When $\theta = \pi/2$, the eigenmode.

$$\begin{pmatrix} S & -iD & 0 \\ iD & S-n^2 & 0 \\ 0 & 0 & P-n^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0 \quad \text{—— (i)}$$

One obvious way of solving this equ^n is to have $P-n^2=0$,

or $\omega^2 = \Pi_e^2 + k^2c^2$ —— (ii)

with the eigenvector $(0, 0, E_z)$. Since the wave-vector now points in the x- direction, this is clearly a transverse wave polarized with its electric field parallel to the equilibrium magnetic field. Particle motions are along the magnetic field, so the mode dynamics are completely unaffected by this field. Thus, the wave is identical to the electromagnetic plasma wave found previously in an unmagnetized plasma. This wave is known as the ordinary, or O-, mode.

The other solution to equ^n (1) is obtained by setting the 2×2 determinant involving the x- and y- components of the electric field to zero. The dispersion relation reduces to

$$n^2 = \frac{RL}{S} \quad \text{—— (3)}$$

with the associated eigenvector $E_x(1, -iS/D, 0)$. Let us, first of all, search for the cutoff frequencies, at which n^2 goes to zero.

According to equ^n (3), These frequencies are the roots of $R=0$ and $L=0$. In fact, we have already solved these equ^{n}s. There are two cutoff frequencies, ω_1 and ω_2, ~~which are specified by~~.

Let us next search for the resonant frequencies at which n^2 goes the infinity. According to equ^n (3). The resonant frequencies are solution of

$$S = 1 - \frac{\pi_e^2}{\omega^2 - \Omega_e^2} - \frac{\pi_i^2}{\omega^2 - \Omega_i^2} = 0 \quad \text{--- (4)}$$

The root of this equ^{n}s can be obtained as follows. First, we note that if the first two terms are equated to zero, we obtain $\omega = \omega_{UH}$, where

$$\omega_{UH} = \sqrt{\pi_e^2 + \Omega_e^2}. \quad \text{--- (5)}$$

If this frequency is substituted into the third term, the result is far less than unity. We conclude that ω_{UH} is a good approximation to one of the roots of equ^n (4). To obtain the second root, we make use of the fact that the product of the square of the roots is

$$\Omega_e^2\Omega_i^2 + \pi_e^2\Omega_i^2 + \pi_i^2\Omega_e^2 \simeq \Omega_e^2\Omega_i^2 + \pi_i^2\Omega_e^2 \quad \text{--- (6)}$$

We, thus obtain $\omega = \omega_{LH}$, where

$$\omega_{LH} = \sqrt{\frac{\Omega_e^2\Omega_i^2 + \pi_i^2\Omega_e^2}{\pi_e^2 + \Omega_e^2}} \quad \text{--- (7)}$$

The first resonant frequency, ω_{UH}, is greater than the electron cyclotron or plasma

t. No. Page No.

frequencies, and is called the upper hybrid frequency. The 2nd resonant frequency, ω_{LH}, lies between the electron and ion cyclotron frequencies, and is called the lower hybrid frequency.

Unfortunately, there is no simple explanation of the origins of the two hybrid resonances in terms of the motions of individual particles.

At low frequencies, the mode in question reverts to the compressional Alfven wave discussed previously. Note that the shear-Alfven wave does not propagate perpendicular to the magnetic field.

Using the above information, and the easily demonstreated fact that

$$\omega_{LH} < \omega_2 < \omega_{UH} < \omega_1, \qquad (8)$$

we can deduce that the dispersion curve for the mode in question takes the form sketched in figure. The lowest frequency branch corresponds to the compressional-Alfven wave. The other two branches constitute the extraordinary, or x^-, wave. The upper branch is basically a linearly polarized electromagnetic wave, somewhat modified by the presence of the plasma. This branch corresponds to a wave which propagates in the

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absence of an equilibrium magnetic field. The lowest branch corresponds to a wave which does not propagate in the absence of an equilibrium field. Finally, the middle branch corresponds to a wave which converts into an electrostatic plasma wave in the absence of an equilibrium magnetic field.

Dispersion for a wave propagating perpendicular to the magnetic field in a magnetized plasma.

Wave propagation at oblique angles is generally more complicated than propagation parallel or perpendicular to the equilibrium magnetic field, but does not involve any new physical effects.